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MODELLING AND PROCESSING FOR CONDITION MONITORING AND TECHNOLOGIES ANALYSIS

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In this article the most general model that can be used is one in which the basic parameters we need in equations for describing advanced vibratory processing machinery are estimated, based upon obtained exact solutions for a relatively few second-order differential equations; the more important of these exact solutions are listed. They are exact in the sense that the solution is given in an expression that can be evaluated numerically to any desired degree of accuracy. The model of VSM that experiences simple harmonic motion and the governing equations sufficient to describe the behavior of the physical system adequately are used as the model for parameter estimation and to study equipment design problems.

Keywords: condition monitoring; modeling and vibratory processing; machinery analysis; optimization in CM.

In the field of the mechanical design and application of the vibration forces arising from any type of dynamic excitations for technologies mathematical concepts involved in theoretical analysis of the method are described in the papers [1, 2]. Process motivated by the acceleration of a deck undergoing harmonic motion occurs when the deck surface has both longitudinal and lateral tilts and follows a displacement pattern in directions: (1) not having a lateral tilt, and (2) defined by a lateral tilt. Interesting surveys of the entire realm of the occurrence of new phenomena are predicted, are explainable, and commercially available with particular application associated with the design and operation of sufficiently new equipment. Every phase of the above mentioned process is affected by the goals which are established, particularly with regard to that phase upon (1) that has been studied and the greatest progress has been made in treating vibration technologies. This item describes largely in mathematical terms certain of the more important features of vibration theory likely to be encountered in practice, and provides furthest information concerning features of the methods of analysis which find ready application.

The systems treated are systems with a finite number of degree-of-freedom which can be defined by a finite number of simultaneous ordinary differential equations. It should be pointed out that vibration processes are based on the velocity transfer from a vibrating deck to the flow of material on it due to the existence of a vibrating force. In practice, this requires the intimate contact of the deck and particles for some period of time, during which equilibrium is approached as velocity transfer proceeds. Process synthesis represents the inventive aspect of process design. Because of to maximize efficiency, the contacting time in a particular stage may not be long enough. It is usually desirable to use vibration defined by the acceleration amplitudes of the deck expressed as a dimensionless multiple of the gravitational acceleration in a range from 6 to 10g. At such values of vibration acceleration particles will be intensively tossed upwards after a brief contact with the deck.

The plane of the X and Z axes is a plane of symmetry. The vibration force is acting along the X axis at the angle of vibration to its direction lengthwise and to the plane of the deck. The angle of lateral tilt of the deck surface (angle of ε) can not be varied. The key difference over the design of machinery is the following: there is no inclination in direction of the acting force at the angle of ε .

Suppose that X, Y, Z fixed in the deck is a convenient set of axes with its origin at the point of material loading. The Y axis is perpendicular to the plane of the deck; the X and Z axes are oriented along and transverse. The direction of the motion of the deck and its change of position are characterized by the angles of vibration β , longitudinal and lateral tilts α and ε , respectively. The values of these angles can be varied as follows

$$\begin{gathered} -\pi/2 < \alpha < \pi/2, \\ 0 < \beta < \pi/2 - \alpha, \\ 0 < \varepsilon < \pi/2. \end{gathered} \tag{1.1}$$

The initial conditions are

when $t_0 = t_0^*$: $x = 0,\ y = 0,\ z = 0,$

$$\frac{dx}{dt} = \left.\frac{dx}{dt}\right|_0^* \equiv (v_x)_0^*,\ \frac{dy}{dt} = \left.\frac{dy}{dt}\right|_0^* \equiv (v_y)_0^*,\ \frac{dz}{dt} = \left.\frac{dz}{dt}\right|_0^* \equiv (v_z)_0^*.$$

The differential equations of particle motion in the X, Y, Z directions are

$$\begin{gathered} \frac{d^2x}{dt^2} = -g\sin\alpha + A\omega^2\cos\beta\sin\omega t, \\ \frac{d^2y}{dt^2} = (-g\cos\alpha + A\omega^2\sin\beta\sin\omega t)\cos\varepsilon, \\ \frac{d^2z}{dt^2} = (g\cos\alpha - A\omega^2\sin\beta\sin\omega t)\sin\varepsilon, \end{gathered} \tag{1.2}$$

where g is the gravitational acceleration; A is the amplitude of the displacement of the deck; ω is the angular frequency of the simple harmonic motion.

By integrating once and twice, respectively, the equations are written as follows

$$\frac{dx(t)}{dt} = -g(t-t_0^*)\sin\alpha - A\omega\cos\beta(\cos\omega t - \cos\omega t_0^*) + \left.\frac{dx}{dt}\right|_0^*,$$

$$\begin{aligned} x(t) = -g\frac{(t-t_0^*)^2}{2}\sin\alpha - A\cos\beta(\sin\omega t - \sin\omega t_0^*) + \\ +A\omega(t-t_0^*)\cos\beta\cos\omega t_0^* + (v_x)_0^*(t-t_0^*), \end{aligned} \tag{1.3}$$

$$\frac{dy(t)}{dt} = -g(t-t_0^*)\cos\alpha\cos\varepsilon - A\omega\sin\beta\cos\varepsilon(\cos\omega t - \cos\omega t_0^*) + \left.\frac{dy}{dt}\right|_0^*,$$

$$\begin{aligned} y(t) = -g\frac{(t-t_0^*)^2}{2}\cos\alpha\cos\varepsilon - A\sin\beta\cos\varepsilon(\sin\omega t - \sin\omega t_0^*) + \\ +A\omega(t-t_0^*)\sin\beta\cos\varepsilon\cos\omega t_0^* + (v_y)_0^*(t-t_0^*), \end{aligned} \tag{1.4}$$

$$\frac{dz(t)}{dt} = -g(t-t_0^*)\cos\alpha\sin\varepsilon + A\omega\sin\beta\sin\varepsilon(\cos\omega t - \cos\omega t_0^*) + \left.\frac{dz}{dt}\right|_0^*,$$

$$\begin{aligned} z(t) = g\frac{(t-t_0^*)^2}{2}\cos\alpha\sin\varepsilon + A\sin\beta\sin\varepsilon(\sin\omega t - \sin\omega t_0^*) - \\ -A\omega(t-t_0^*)\sin\beta\sin\varepsilon\cos\omega t_0^* + (v_z)_0^*(t-t_0^*). \end{aligned} \tag{1.5}$$

The phase angles at the time of flight initiation of the particle over the vibrating deck and of its downfall on the deck, respectively, are

$$\delta_0^* = \omega t_0^*,\ \delta_n^* = \omega t_n^*.$$

For the period of particle flight over the deck defined by $T = pT_0$, where T_0 is the time interval to complete one cycle in the deck vibration, the following expression can be written

$$\delta_n - \delta_0^* = 2\pi p. \tag{1.6}$$

The velocity of particle fall down on the surface of the deck $(v_y)_n$ and the velocity of particle reflection and flight initiation $(v_y)_0^*$ are known, in accordance with Newton's theory of impact, as

$$R = \frac{(v_y)_0^*}{(v_y)_n}. \tag{1.7}$$

Using expression (1.6), the following is obtained

$$\omega t_n = 2\pi p + \omega t_0^*,\ \ t = t_n = \frac{2\pi p}{\omega} + \omega t_0^*. \tag{1.8}$$

The Eq. (1.4) give the following

$$(v_y)_n = -g(t_n - t_0^*)\cos\alpha\cos\varepsilon - A\omega\sin\beta\cos\varepsilon(\cos\omega t_n - \cos\omega t_0^*) + (v_y)_0^*. \tag{1.9}$$

Since

$$\cos\omega t_n = \cos(2\pi p + \omega t_0^*) = \cos\omega t_0^*,$$

this gives

$$(v_y)_n = -g(t_n - t_0^*)\cos\alpha\cos\varepsilon + (v_y)_0^*.$$

Using relation (1.8), the following result is obtained

$$(v_y)_n = -g\left(\frac{2\pi p}{\omega}\right)\cos\alpha\cos\varepsilon + (v_y)_0^*.$$

In accordance with equality (1.7) the preceding equation can be written

$$\frac{(v_y)_0^*}{R} = -g\left(\frac{2\pi p}{\omega}\right)\cos\alpha\cos\varepsilon + (v_y)_0^*$$

or

$$(v_y)_0^*\left(1+\frac{1}{R}\right) = \left(\frac{2\pi pg}{\omega}\right)\cos\alpha\cos\varepsilon,$$

so that the equation can be expressed as

$$(v_y)_0^* = \left(\frac{2\pi pg}{\omega}\right)\left(\frac{R}{1+R}\right)\cos\alpha\cos\varepsilon. \qquad (1.10)$$

Eq.(1.8) makes $t_n - t_0^* = \dfrac{2\pi p}{\omega}$, substituting this relation, Eq (1.4) becomes

$$-g\left(\frac{2\pi p}{\omega}\right)^2\frac{\cos\alpha\cos\varepsilon}{2} - A\sin\beta\cos\varepsilon(\sin\omega t - \sin\omega t_0^*) +$$

$$+A\omega\cos\varepsilon\frac{2\pi p}{\omega}\sin\beta\cos\omega t_0^* + \frac{2\pi p}{\omega}(v_y)_0^* = 0.$$

Since $\cos\omega t_0^* = \cos\delta_0^*$ and $\sin\omega t_n = \sin\omega t_0^*$, dividing $2\pi p$ and $A\sin\beta\cos\varepsilon$, the following equation is obtained

$$\frac{-g\cos\alpha}{A\omega^2\sin\beta}\pi p + \cos\delta_0^* + \frac{(v_y)_0^*}{A\omega\sin\beta\cos\varepsilon} = 0, \qquad (1.11)$$

where $\omega_0 = \dfrac{A\omega^2\sin\beta}{g\cos\alpha}$ is the coefficient of dynamic load.

The phase angle of flight initiation of the particle over the vibrating deck can be calculated by the formula

$$\cos\delta_0^* = \frac{\pi p}{\omega_0}\left(1 - \frac{2R}{1+R}\right)$$

or

$$\cos\delta_0^* = \frac{\pi p}{\omega_0}\cdot\frac{1-R}{1+R}. \qquad (1.11a)$$

The particle displacement in the X direction at the time interval to complete one cycle of the particle flight, using the appropriate Eq. (1.3), is

$$S_X = x(t) = -g\frac{(t_n - t_0^*)^2}{2}\sin\alpha - A\cos\beta(\sin\omega t_n - \sin\omega t_0^*) +$$
$$+A\omega(t_n - t_0^*)\cos\beta\cos\omega t_0^* + (v_x)_0^*(t - t_0^*).$$

Since $t_n = \frac{\delta_n}{\omega}$, $t_0^* = \frac{\delta_0^*}{\omega}$ and $t_n - t_0^* = \frac{\delta_n - \delta_0^*}{\omega}$, this can be written

$$S_X = -\frac{0.5g}{\omega^2}\sin\alpha(\delta_n - \delta_0^*)^2 + A\cos\beta(\sin\delta_n - \sin\delta_0^*) +$$
$$+A\cos\beta\cos\delta_0^*(\delta_n - \delta_0^*) + \frac{(v_x)_0^*(\delta_n - \delta_0^*)}{\omega}.$$

The phase angles at time of flight initiation of the particle over the vibrating deck and of its downfall on the deck are related as follows

$$\delta_n = 2\pi p + \delta_0^*.$$

Using this relation,

$$S_X = 0.5\left(\frac{2\pi p}{\omega}\right)^2 g\sin\alpha + A\cos\beta\left[\sin(2\pi p + \delta_0^*) - \sin\delta_0^*\right] +$$
$$+2\pi pA\cos\beta\cos\delta_0^* + \frac{2\pi p}{\omega}(v_x)_0^*.$$

Since

$$\sin(2\pi p + \delta_0^*) - \sin\delta_0^* = 0,$$
$$\cos\delta_0^* = \frac{g\cos\alpha}{A\omega^2\sin\beta}\pi p - \frac{(v_x)_0^*}{A\omega\sin\beta\cos\varepsilon},$$

from the previous equation

$$S_X = -\frac{g}{2}\left(\frac{2\pi p}{\omega}\right)^2\sin\alpha + 2\pi pA\cos\beta\left(\frac{g\cos\alpha}{A\omega^2\sin\beta}\pi p - \frac{(v_y)_0^*}{A\omega^2\sin\beta\cos\varepsilon}\right) +$$
$$+\frac{2\pi p}{\omega}(v_x)_0^* = \frac{g}{2}\left(\frac{2\pi p}{\omega}\right)^2\left(\frac{\cos\alpha}{\tan\beta} - \sin\alpha\right) + \frac{2\pi p}{\omega}\left((v_x)_0^* - \frac{(v_y)_0^*}{\cos\varepsilon\tan\beta}\right).$$

Letting $\frac{\cos\alpha}{\tan\beta} - \sin\alpha = \frac{\cos(\alpha+\beta)}{\sin\beta}$, the preceding equation can be written

$$S_X = \frac{g}{2}\left(\frac{2\pi p}{\omega}\right)^2\frac{\cos(\alpha+\beta)}{\sin\beta} + \frac{2\pi p}{\omega}\left((v_x)_0^* - \frac{(v_y)_0^*}{\cos\varepsilon\tan\beta}\right). \qquad (1.12)$$

Eq.(1.10) gives

$$(v_y)_0^* = \frac{2\pi pg}{\omega}\cdot\frac{R}{R+1}\cos\alpha\cos\varepsilon.$$

Similarly, v_x^{**} becomes

$$v_x^{**} = (v_x)_0^* = (1-\lambda^{-1})\frac{2\pi p}{\omega} g \sin\alpha, \tag{1.13}$$

where λ is the coefficient of instantaneous friction. Substituting these terms $(v_x)_0^*$ and $(v_y)_0^*$ into (1.12), the following equation is obtained

$$S_X = \frac{g}{2}\left(\frac{2\pi p}{\omega}\right)^2 \left(\frac{\cos(\alpha+\beta)}{2\sin\beta} - \sin\alpha\frac{1-\lambda}{\lambda} - \frac{R}{R+1}\cdot\frac{\cos\alpha}{\tan\beta}\right).$$

The mathematical equation of particle velocity for motion in the direction of the X-axis

$$V_X = \frac{S_X\omega}{2\pi p} = \frac{2\pi pg}{\omega}\left[\cos\alpha\cot\beta\left(\frac{1}{2} - \frac{R}{R+1}\right) - \sin\alpha\left(\frac{1}{2} - \frac{1-\lambda}{\lambda}\right)\right].$$

Simplifying this, the equation can be reduced to

$$V_X = \frac{\pi pg}{\omega}\left(\frac{1-R}{1+R}\cos\alpha\cot\beta - \frac{2-\lambda}{\lambda}\sin\alpha\right). \tag{1.14}$$

Since Eqs. (1.3) and (1.5) are of the same form, the solutions are the same except for the directions of particle motion. The solution of Eq.(1.5) is of the form in which the particle displacement in the Z direction is considered

$$S_Z = z(t_n) =$$
$$= \left[0.5g(t_n - t_0^*)^2\cos\alpha + A\sin\beta(\sin\omega t_n - \sin\omega t_0^*) - A\omega(t_n - t_0^*)\sin\beta\cos\omega t_0^*\right]\cos\varepsilon +$$
$$+ (v_z)_0^*(t_n - t_0^*).$$

Using the relations of Eqs. (1.6) and (1.8), this equation can be expressed as follows

$$S_Z = \left[0.5g\left(\frac{2\pi p}{\omega}\right)^2\cos\alpha - 2\pi pA\sin\beta\cos\delta_0^*\right]\sin\varepsilon + \frac{2\pi p}{\omega}(v_z)_0^*. \tag{1.15}$$

It is assumed that

$$\frac{(v_x')_n}{(v_x)_n} = 1-\lambda; \ \frac{(v_z')_n}{(v_z)_n} = 1-\lambda. \tag{1.16}$$

After a determined interval of time $\Delta t = \dfrac{2\pi p}{\omega}$, the increment of particle velocity toward the lateral direction is

$$\Delta(v_z)_n = (v_z')_n - (v_z)_0^* = (v_z)_n(1-\lambda) - (v_z)_0^*,$$

where $(v_z)_0^*$ is the projection of particle velocity on the Z axis at the time of flight initiation. Using Eq. (1.5), the following is obtained

$$\Delta(v_z)_n = \left[g(t-t_0^*)\cos\alpha + A\omega\sin\beta(\cos\omega t - \cos\omega t_0^*)\right]\sin\varepsilon(1-\lambda) +$$
$$+(v_z)_0^*(1-\lambda)-(v_z)_0^* = \frac{2\pi p}{\omega} g\cos\alpha\sin\varepsilon(1-\lambda)-\lambda(v_z)_0^*. \quad (1.17)$$

Projection of particle velocity on the Z axes is

$$v_z^{**} = (v_z)_0^* = -(1-\lambda^{-1})\frac{2\pi p}{\omega} g\cos\alpha\sin\varepsilon.$$

Substitution this relation into Eq. (1.15) gives

$$S_Z = \left[\frac{g}{2}\left(\frac{2\pi p}{\omega}\right)^2\cos\alpha - 2\pi pA\sin\beta\cos\delta_0^*\right]\sin\varepsilon + \frac{(2\pi p)^2}{\omega^2}\times$$
$$\times\frac{1-\lambda}{\lambda} g\cos\alpha\sin\varepsilon = \left[g\cos\alpha\left(\frac{2\pi p}{\omega}\right)^2\left(\frac{1}{2}+\frac{1-\lambda}{\lambda}\right)-2\pi pA\sin\beta\cos\delta_0^*\right]\sin\varepsilon. \quad (1.18)$$

Since $\cos\delta_0^*$ is determined by (1.11a), Eq.(1.18) can be written

$$S_Z = \left(2g\cos\alpha\frac{\pi^2 p^2}{\omega^2}\cdot\frac{2-\lambda}{\lambda} - 2g\cos\alpha\frac{\pi^2 p^2}{\omega^2}\cdot\frac{1-R}{1+R}\right)\sin\varepsilon =$$
$$= 2g\cos\alpha\sin\varepsilon\frac{\pi^2 p^2}{\omega^2}\left(\frac{2-\lambda}{\lambda}-\frac{1-R}{1+R}\right). \quad (1.19)$$

which can be reduced to

$$V_Z = \frac{S_Z\omega}{2\pi p} = \frac{\pi pg}{\omega}\left(\frac{2-\lambda}{\lambda}-\frac{1-R}{1+R}\right)\cos\alpha\sin\varepsilon. \quad (1.20)$$

Applicable mathematical expressions

The equations (1.14) and (1.20) for the particle velocity in its movement in the longitudinal and lateral directions are the basic equations in the theory of vibration separation. For the purpose of studying the effects of vibration these equations can be transformed to other forms required for particular applications.

At a specific instant of the phase angle corresponding to the particle flight initiation motivated by the maximum value of the acceleration of vibration, when $\cos\delta_0^* = 1$, from the expression (1.11a)

$$p = \frac{1}{\pi}\left(\frac{1+R}{1-R}\right)\omega_0.$$

In this case, if it is assumed that

$$q = \frac{\lambda}{2-\lambda}\cdot\frac{1-R}{1+R},$$

is the parameter of separation, Eqs. (1.14) and (1.20) can be converted to the form

$$V_X = A\omega\left(\cos\beta - \frac{\sin\beta\tan\alpha}{q}\right), \quad (1.21)$$

$$V_Z = A\omega \sin\beta \sin\varepsilon\left(\frac{1}{q}-1\right). \tag{1.22}$$

Dividing Eq. (1.22) by Eq.(1.21)

$$\frac{dz}{dx}=\frac{V_Z}{V_X}=\frac{\sin\beta\sin\varepsilon\left(\frac{1}{q}-1\right)}{\cos\beta-\frac{\sin\beta\tan\alpha}{q}}=\frac{(1-q)\sin\varepsilon}{q\cot\beta-\tan\alpha}. \tag{1.23}$$

The Eq. (1.23) can be used to construct the trajectory of particle movement. This is done as illustrated in Fig.1 of [7]: resulting trajectories are drawn by broken-dotted lines through a point taken near the origin of the separation load. The directions of movement of the components are such as to take the trajectories away from the origin. The angle of particle orientation in its movement is indicated by the formula

$$\tan\psi_I=\frac{(1-q)\sin\varepsilon}{q\cot\beta-\tan\alpha}. \tag{1.24}$$

The XZ plane of the deck in is a plane of symmetry. Each parallel deck with its longitudinal and lateral axis in the X and Z directions, respectively, has the length l and the width b of the deck, respectively. So $x=b/\tan\psi$, $z=l\tan\psi$. Separation ability is defined analytically as follows

$$D_X=\left.\frac{dx}{dq}\right|_{z=b}=b\frac{d}{dq}\left(\frac{1}{\tan\psi}\right),$$

$$D_z=-\left.\frac{dz}{dq}\right|_{x=1/2}=-\frac{1}{2}\frac{d}{dq}\left(\tan\psi\right).$$

Solving Eq.(1.23)

$$D_{X_I}=b\frac{d}{dq}\left(\frac{q\cot\beta-\tan\alpha}{(1-q)\sin\varepsilon}\right)=\frac{b}{\sin\varepsilon}\cdot\frac{\cot\beta-\tan\alpha}{(1-q)^2}, \tag{1.25}$$

$$D_{Z_I}=-\frac{1}{2}\frac{d}{dq}\left(\frac{(1-q)\sin\varepsilon}{q\cot\beta-\tan\alpha}\right)=\frac{1}{2}\sin\varepsilon\frac{\cot\beta-\tan\alpha}{(q\cot\beta-\tan\alpha)^2}. \tag{1.26}$$

Conclusions

1. In the operation of modeling and processing for CM, there are several factors to be considered that are important regardless of the particular methodology. The most important consideration involves using the basic equations for evaluating the design parameters technologies.
2. Mathematical model method for parameter estimation is used as the theoretical basis for the description of processing for CM with the second-order differential equations whose solutions are obvious from adequate analytical models.

3. The method has been applied extensively in studies of the governing equations and some of the general mathematical expressions thereby obtained are given here, both as for future evaluations and as a collection of useful relations which find ready application in practice.
4. The solution procedure involves representation of the function behavior in the form of response curves versus parameters. The response curves for a particular system acted on by a harmonic exciting force may be derived from the obtained equations for the condition monitoring.

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ И МОНИТОРИНГ ВИБРАЦИОННЫХ ТЕХНОЛОГИЙ

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В статье проводится математическое моделирование и анализ динамики вибрирующих систем в целях разработки принципиальной схемы, значений параметров, наиболее рациональной динамической структуры и элементов оптимальной конструкции аппаратов вибрационного действия. В результате разработаны научные основы и установлены основные закономерности процессов, позволяющие производить расчет основных параметров. Получены теоретические уравнения и ряд формул для расчета в зависимости от режима и вида колебаний и математическая модель процессов.

Ключевые слова: мониторинг; моделирование; математический анализ; вибрационные технологии; оптимизация параметров и конструкций.